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Unification in models with replicated gauge groups

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Abstract: We examine unification of gauge couplings in four dimensional renormalizable gauge theories inspired by the latticized (deconstructed) SM or MSSM in five dimensions. The models are based on replicated gauge groups, spontaneously broken to the diagonal subgroup. The analysis is performed at the one-loop level, with the contribution from the heavy vector bosons included, and compared with the analogous results in the SM or MSSM. Unification at the diagonal breaking scale is compared with the unification above that scale proposed by Arkani-Hamed, Cohen and Georgi. We find that in the considered class of extensions of the MSSM(SM) unification is possible for a wide range of unification scales and with a similar accuracy as in the MSSM(SM).

Keywords: Beyond Standard Model, GUT.

Contents

1. Introduction

In recent papers it has been demonstrated [1, 2] that there exists an interesting class of renormalizable four-dimensional gauge theories that predict the same infrared physics as five-dimensional gauge theories. The gauge group of the four-dimensional theory is a product of N copies of $\mathrm{SU}(k)$ group and is spontaneously broken at some scale v to the diagonal subgroup by a set of link-Higgs fields in the bifundamental representations $(\mathbf{k_n}, \bar{\mathbf{k}}_{\mathbf{n+1}})$ of the $\mathrm{SU}(k)_n$ group. In addition to the massless gauge bosons of the unbroken $\mathrm{SU}(k)$, the theory predicts the existence of massive gauge bosons transforming in the adjoint representations of the unbroken diagonal $\mathrm{SU}(k)$. In the limit of large-N their mass spectrum and interactions are identical to that of the Kaluza Klein (KK) modes of the compactified 5d $\mathrm{SU}(k)$ gauge theory (after proper mapping of the parameters of the two theories; in particular the scale v of the symmetry breaking has to be of the same magnitude as the UV cut-off of the 5d theory).

Thus, the four-dimensional theory at distances *larger* than $1/v$ predicts the phenomena that are usually attributed to extra dimensions (the appearance of massive KK modes, deviations from the Coulomb law). The obvious advantage of the four-dimensional set-up is its renormalizability and, moreover, the possibility to discuss physics also at distances *shorter* than $1/v$ where the full gauge symmetry is restored. These properties make this class of 4d theories interesting in its own, not just as an UV completion of 5d non-renormalizable theories. Many possibilities for constructing four-dimensional models inspired by extra dimensional gauge theories open up, and a number of examples have already been presented [3, 4].

In this paper we examine the running of gauge couplings in a model based on N-fold replication of $G_{\rm SM} \equiv \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ gauge group. The theory is parametrized by N sets of gauge couplings $\alpha_1^{(n)}, \alpha_2^{(n)}, \alpha_3^{(n)}$ $(n = 1, \ldots, N)$. At some scale v the full gauge symmetry $[G_{\rm SM}]^N$ is broken to the diagonal subgroup, which we identify with the SM or MSSM gauge group, with the gauge couplings $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3$. Above the diagonal breaking scale it is natural to discuss the evolution of the couplings $\alpha_i^{(n)}$ of each single factor of $[G_{\rm SM}]^N$, as the full gauge symmetry is restored. Moreover, we can treat all the fields as massless. Below the scale v one could attempt to construct effective theories at each stage of decoupling a single massive gauge boson but this is problematic, since the full gauge group is broken to the diagonal subgroup in one step. Instead, we can discuss the evolution of the couplings $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3$ in the full spontaneously broken $[G_{\rm SM}]^N$ gauge theory. Below the scale v the gauge couplings of the unbroken $\mathrm{SU}(3) \times \mathrm{SU}(2)$ subgroup can be defined as the full trilinear massless gauge boson effective vertices [5, 6]. For the unbroken U(1) factor we define the gauge coupling as the effective vertex with the massless U(1) gauge boson and two fermions. This allows us to include in the running the threshold effects which are sizeable each time the RGE scale is comparable to the gauge bosons masses. At the scale v the couplings $\tilde{\alpha}_i(v)$ and $\alpha_i^{(n)}(v)$ are connected to each other by the tree-level relation $\frac{1}{\tilde{\alpha}_i(v)} = \sum_n \frac{1}{\alpha_i^{(n)}(v)}$.

We examine the evolution of the gauge couplings at the one-loop level, including all contributions of the massive gauge and Higgs sectors. It is well known that in the SM and MSSM the two-loop effects are important for precision considerations and one may expect the same to be true in the discussed class of models. Working at one-loop we cannot conclude about the absolute precision of unification but we can compare the one-loop unification in the replicated models with the one-loop unification in the SM or MSSM. This is the goal of the present paper and we hope, that our conclusions hold also at the two-loop level.

One can study the gauge coupling unification at the diagonal breaking scale v or above it and these two cases are qualitatively different.[1] The former option is similar to the power-law unification in five dimensions considered in [7, 8, 9], but in our case the calculation is performed in the renormalizable set-up and can be done rigorously. Our conclusions with regard to the unification at the diagonal breaking scale are similar to [9]. First of all, unification is possible for a wide range of $v < 10^{16}$ GeV, but the number of replications required for unification at a given value of v is almost uniquely fixed. In the five-dimensional language it corresponds to a correlation between the cut-off scale and the compactification radius R. Moreover, in order to sizably lower the unification scale we need a large number of replications. In the presence of N gauge bosons the appropriate loop expansion parameter is $N\tilde{\alpha} \equiv \alpha$ rather than $\tilde{\alpha}$ itself. We find that if we try to lower the unification scale below $\sim 10^{10}$ GeV, then α becomes larger than 1 and the perturbative analysis cannot be trusted.

It is well-known that one-loop unification in the SM or MSSM requires threshold

[1]The other logical possibility — unification below the scale v — is very similar to the usual logarithmic running, since the heavy gauge bosons become operative shortly below that scale.

corrections at the unification scale of order $O(10\%)$ or $O(1\%)$, respectively. We find that a similar conclusion holds in the replicated SM, even with lower unification scale. In the replicated MSSM, to lower the unification scale one needs somewhat larger threshold corrections than in the MSSM itself.

Unification above the diagonal breaking scale v has been proposed in ref. [10]. It is a new interesting possibility in the discusssed class of four-dimensional theories, with no counterpart in the five-dimensional theories. In this case unification of the gauge couplings $\alpha_i^{(n)}(M_{\rm GUT}) = \alpha^{(n)}(M_{\rm GUT})$ does not necessarily imply $\alpha^{(n)}(M_{\rm GUT}) = \alpha(M_{\rm GUT})$. The contribution to the beta functions of any two couplings $\alpha_i^{(n)}(Q)$ and $\alpha_i^{(m)}(Q)$ can differ by complete representations of SU(5), e.g. of matter fields. The unification can take place at $M_{\rm GUT}(\gg v)$ well below 10^{16} GeV and remains the prediction of the theory to the same extent as in the SM or MSSM, provided the beta functions for the couplings α_i above v are the same as in SM(MSSM), up to full SU(5) multiplets. Furthermore, the unification takes place for a wide range of N, with $M_{\rm GUT} \approx 10^{16/N} v^{(N-1)/N}$ and in this scenario the unification scale can be lowered sizeably even for small N. Therefore, contrary to the previous case of unification at the diagonal breaking scale, v and N remain independent parameters of the theory. Finally, at least at one-loop, the GUT scale threshold corrections needed for unification remain of similar order of magnitude as in the SM or MSSM, respectively.

In section 2 we define our model. In section 3 we discuss unification at the diagonal breaking scale. Finally, in section 4 we recall the results of ref. [10] to compare them with unification at the diagonal breaking scale.

2. The model

In this section we define our set-up in which we calculate the running of gauge couplings. Our guideline is the assumption that the success of the MSSM unification is not accidental. Therefore we replicate the gauge and Higgs structures, which are crucial for successful unification in the MSSM. We do not replicate the matter fields which affect only the value of the gauge couplings at the unification point and not the unification itself.[2]

The $SU(3) \times SU(2) \times U(1)$ gauge group is replicated N-times. We also add $N-1$ bifundamental link-Higgs fields $\Phi^{(n)}$ which fill out the $(\mathbf{5}, \bar{\mathbf{5}})$ representations of $SU(5) \times SU(5)$. Under the n-th and $(n+1)$-th $SU(3) \times SU(2) \times U(1)$ these link fields split into $\Phi_{\mathbf{33}}$ in $(\mathbf{3}_{1/\sqrt{15}}, \bar{\mathbf{3}}_{-1/\sqrt{15}})$, $\Phi_{\mathbf{22}}$ in $(\mathbf{2}_{-3/2\sqrt{15}}, \bar{\mathbf{2}}_{3/2\sqrt{15}})$, $\Phi_{\mathbf{32}}$ in $(\mathbf{3}_{1/\sqrt{15}}, \bar{\mathbf{2}}_{3/2\sqrt{15}})$ and $\Phi_{\mathbf{23}}$ in $(\mathbf{2}_{-3/2\sqrt{15}}, \bar{\mathbf{3}}_{-1/\sqrt{15}})$. We choose to work with the model in which the first and the N-th $SU(3) \times SU(2) \times U(1)$ group factors are not linked by the Higgs field.

Further, we assume that $\Phi_{\mathbf{33}}^{(n)}$ and $\Phi_{\mathbf{22}}^{(n)}$ link-Higgs fields acquire common vacuum expectation values $\langle \Phi^{(n)} \rangle = v$. In the non-supersymmetric case this can be obtained by choosing the scalar potential for these Higgs fields of the form:

$$V = \sum_n \mathrm{Tr}\left[\left(\Phi^{(n)\,\dagger}\Phi^{(n)} - v^2\right)^2\right] \tag{2.1}$$

[2]Replicating matter fields would make the unified coupling α bigger and perturbativity would be violated for rather small N.

In the supersymmetric case we can construct the model along the lines of [11]. The D-term potential for the link Higgs fields is:

$$V_D = \frac{1}{2} g^2 \sum_{n=1}^{N} \left(\mathrm{Tr} \left[\Phi^{(n)\,\dagger} T^a \Phi^{(n)} - \Phi^{(n-1)} T^a \Phi^{(n-1)\,\dagger} \right] \right)^2 \tag{2.2}$$

where $\Phi^{(0)} = \Phi^{(N)} = 0$ and T^a are the generators of the fundamental representation of the SM gauge group. This potential has a set of minima of the form $\Phi^{(n)} = v_n I$ where v_n are its flat directions. Note that, with the link-Higgs fields alone, no renormalizable superpotential can be constructed. Therefore, the flat directions of the potential have to be lifted by adding soft susy breaking scalar mass terms. Without further explanation we assume that all the VEVs are non-zero and equal: $v_n = v$. Expanding the link-Higgs field around the minimum, $\Phi^{(n)} = vI + \phi^{(n)}$, we find that under the diagonal gauge group each of the link-Higgs fields split into the following three representations: two (real) adjoint $h^a_n = \mathrm{Tr}[T^a(\phi^{(n)} + (\phi^{(n)})^\dagger)]$, $G^a_n = i\mathrm{Tr}[T^a(\phi^{(n)} - (\phi^{(n)})^\dagger)]$ and complex singlets $h_n = \mathrm{Tr}[\phi^{(n)}]$. The adjoints G^a_n are Goldstone bosons and become longitudinal components of the massive gauge bosons. The mass matrix of the physical scalars h^a_n is identical to the one of the gauge bosons and to the mass matrix of the Dirac fermions Ψ^a_n constructed from the superpartners χ^a_n of the gauge bosons and the adjoint fermions coming from the superpartner of the link-Higgs fields $\sqrt{2}\mathrm{Tr}[T^a \Psi^{(n)}]$. Thus, at each gauge boson mass level we have a full $\mathcal{N} = 2$ Yang-Mills supermultiplet. Of course, at one-loop the singlets do not affect the running of the gauge couplings and are irrelevant for our discussion.

The diagonal subgroup, to which the full gauge group $[G_{\mathrm{SM}}]^N$ is broken by the VEVs of the link-Higgs fields, is identified with the gauge group of the SM (MSSM). The SM (MSSM) gauge couplings $\tilde{\alpha}_i$ are given in terms of the couplings $\alpha^{(n)}_i$ of $\mathrm{SU}(3)_n \times \mathrm{SU}(2)_n \times \mathrm{U}(1)_n$ by the formula

$$\frac{1}{\tilde{\alpha}_i(v)} = \sum_{n=1}^{N} \frac{1}{\alpha^{(n)}_i(v)}. \tag{2.3}$$

For the sake of simplicity we assume that the gauge couplings at the scale v are the same for all the N group factors (which is the case in the models of deconstructed dimensions of [1, 2]) so that the formula for the low energy gauge couplings takes the form:

$$\frac{1}{\tilde{\alpha}_i(v)} = \frac{N}{\alpha_i(v)} \tag{2.4}$$

The gauge bosons corresponding to the diagonal $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup remain massless (down to the scale of electroweak symmetry breaking). The remaining gauge boson acquire masses equal to:

$$(M^{(i)}_n)^2 = 16\pi\alpha_i(v) v^2 \sin^2\left(\frac{n\pi}{2N}\right) \tag{2.5}$$

In the supersymmetric case the same masses are acquired by the corresponding Dirac gaugino and the scalar field.

We also replicate N-times the *electroweak* Higgs doublet(s), i.e. we assume the existence of N scalars H_n transforming as $(\mathbf{2})_{1/2}$ of the n-th $\mathrm{SU}(2)\times\mathrm{U}(1)$ group. Next, we introduce trilinear couplings to the link-Higgs fields of the form:

$$\mathcal{L} \sim m_n (H_n^{\dagger}\Phi^{(n)} H_{n+1}) + \mathrm{H.c.} \tag{2.6}$$

where m_n are dimensionful couplings. The expectation value of the link-Higgs field produces a mass matrix for the electroweak Higgs. We choose the parameters m_n so that the tower of Higgs boson masses is the same as that of SU(2) gauge bosons masses:

$$(M_n)^2 = 16\pi\alpha_2(v) v^2 \sin^2\left(\frac{n\pi}{2N}\right) \tag{2.7}$$

In the supersymmetric case we need $2N$ chiral supermultiplets $H_n^{(k)}$ with the weak hypercharges $Y = \pm 1/2$. The mass tower similar as in the non-supersymmetric case can be obtained by choosing the superpotential of the form: $W \sim \sum_n (\epsilon H_n^{(2)} \Phi_{\mathbf{22}}^{(n)} H_{n+1}^{(1)} + \mu_n H_n^{(1)} H_n^{(2)})$. Again, by tuning the parameters of the superpotential the tower of electroweak Higgs mass eigenstates with masses given by (2.7) can be constructed.

The matter fields, which are assumed to be those of the SM or MSSM, are not replicated and transform under one, say the first, of the G_{SM} groups.[3] The field ϕ in represantation r of $G_{1\,\mathrm{SM}}$ and coupled to its gauge boson with a couplings g, transforms under the diagonal group as representation r and its coupling is coupling $\tilde{g} = g/\sqrt{N}$. In addition, ϕ couples to the whole tower of massive gauge bosons, and the coupling is given by $\tilde{g} = \sqrt{2}\cos\left(\frac{n\pi}{2N}\right)\frac{g}{\sqrt{N}}$.

3. Unification at the diagonal breaking scale

In order to define what we understand by 'successful unification' let us first recall the one-loop renormalization group equations in the SM and MSSM. At one-loop the gauge couplings $\tilde{\alpha}_i$ of the three group factors of G_{SM} run according to the equations:

$$\frac{1}{\tilde{\alpha}_i(Q)} = \frac{1}{\tilde{\alpha}_i(M_Z)} - \frac{b_0^{(i)}}{2\pi}\ln\left(\frac{Q}{M_Z}\right) + \delta_i \tag{3.1}$$

Here, $1/\tilde{\alpha}_i(M_Z) = (58.98 \pm 0.04,\ 29.57 \pm 0.03,\ 8.40 \pm 0.14)$ [12] are the experimental values of the gauge couplings at the Z^0-pole and $b_0^{(i)}$ are the one-loop coefficients of the relevant beta-functions. In the limit in which all (s)particles are massless, they read $b_0 = (\frac{1}{10} + \frac{4}{3}N_g, -\frac{43}{6} + \frac{4}{3}N_g; -11 + \frac{4}{3}N_g)$ in the SM and $b_0 = (3/5 + 2N_g; -5 + 2N_g; -9 + 2N_g)$ in the MSSM, where N_g is the number of generations. Threshold corrections (e.g. from heavy GUT gauge bosons [13]) are represented by the parameters δ_i.

In the bottom-up approach one can speak about the gauge coupling unification if in some range of scales Q the couplings defined by eq. (3.1) with, in general, Q-dependent

[3] Note that the running of the $\alpha_i^{(1)}$ gauge couplings above the scale v is then different from the running of the remaining $N-1$ couplings $\alpha_i^{(n)}$. In conseqence, the relation $\alpha_i^{(1)} = \alpha_i^{(2)} = \ldots = \alpha_i^{(N)}$ is not preserved above v.

$\delta_i(Q)$ can take a common value $\alpha_i(Q) = \alpha_{\rm GUT}$ for reasonably small values of $\delta_i(Q)$ (compared to $\alpha_{\rm GUT}^{-1}$).[4] The condition for the unification can be succintly written as

$$\epsilon_{ijk}\left(\frac{1}{\tilde{\alpha}_i(M_Z)} + \delta_i\right)(b_0^{(j)} - b_0^{(k)}) = 0 \tag{3.2}$$

Putting in the experimental values for $\alpha_i(M_Z)$ and the beta-function coefficients we get:

$$\begin{aligned} -41.1 + 3.8\delta_1 - 11.1\delta_2 + 7.3\delta_3 &= 0 \quad \text{(SM)} \\ -0.9 + 4\delta_1 - 9.6\delta_2 + 5.6\delta_3 &= 0 \quad \text{(MSSM)} \end{aligned} \tag{3.3}$$

Wee see, that to achieve the gauge coupling unification at the one-loop level we need the threshold corrections δ_i to be of order 10% $\alpha_{\rm GUT}^{-1}$ in the SM, while in the MSSM we need only δ_i ∼1% $\alpha_{\rm GUT}^{-1}$. The latter number suggests that the gauge coupling unification in the MSSM can be very succesful. Indeed, in precision calculations [14, 15] non-negligible two-loop corrections are approximately cancelled out by superpartner threshold corrections for superpartner masses O(1 TeV) [15].

With the threshold corrections of the right order of magnitude, the unification scale can be estimated from the equation:

$$\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} - \frac{1}{2\pi}\left(b_0^{(1)} - b_0^{(2)}\right)\ln\left(\frac{M_{\rm GUT}}{M_Z}\right) + (\delta_1 - \delta_2) = 0 \tag{3.4}$$

For the sake of concreteness, in this paper we shall always assume that $\delta_1 = \delta_2 = 0$ and that all threshold corrections are accounted for by δ_3 (thus, the unification point is assumed to be where α_1 and α_2 intersect). Putting in the experimental numbers and the beta-function coefficients we get $M_{\rm GUT} \approx 1 \times 10^{13}\,$GeV in the SM and $M_{\rm GUT} \approx 2 \times 10^{16}\,$GeV in the MSSM.

We now turn to the models with replicated gauge groups. In this section we investigate the possibility that unification occurs at the scale at which the $[\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)]^N$ group is broken down to its diagonal subgroup by the link-Higgs boson VEV's. In practice, it is convenient to choose for this scale the mass of the heaviest gauge boson $M_{\rm GUT} \equiv M_{N-1} = 2gv\sin((N-1)\pi/2N)$. The massive gauge bosons corresponding to broken symmetries transform in the adjoint representation of the unbroken group and thus contribute to the running of the SM (MSSM) coupling constants. Working at the one-loop level, we include them in the step-function approximation, in which only the particles lighter than the actual scale Q contribute to the coefficients of the beta-functions. For the gauge couplings defined as full vertices, the step-function decoupling procedure is justified in appendix A. The one-loop renormalization group equations for scales Q such that $M_n^{(i)} < Q < M_{n+1}^{(i)}$ are

$$\frac{1}{\tilde{\alpha}_i(Q)} = \frac{1}{\tilde{\alpha}_i(M_Z)} - \frac{b_0^{(i)}}{2\pi}\ln\left(\frac{Q}{M_Z}\right) - \frac{\tilde{b}^{(i)}}{2\pi}\ln\left(\frac{Q^n}{M_1^{(i)}\cdots M_n^{(i)}}\right) + \delta_i \tag{3.5}$$

The beta-function coefficients for the massive gauge sector are (see appendix A) $\tilde{b} = (1/10, -41/6, -21/2)$ in the SM and $\tilde{b} = (3/5, -3, -6)$ in the MSSM. Assuming, that

[4]Whether there exists a unified model able to provide such values of $\delta_i(Q)$'s is a different question.

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$M_{\rm GUT}$ (GeV)	v (GeV)	N	F	$\tilde{\alpha}_{\rm GUT}^{-1}$	$\delta_3/\tilde{\alpha}_{\rm GUT}^{-1}$	α
1.2×10^5	1.4×10^4	62	40.2	47.6	-5.4%	1.30
1.1×10^6	1.3×10^5	57	36.8	45.6	-5.1%	1.25
1.3×10^8	1.8×10^7	46	29.3	41.3	-4.4%	1.11
1.1×10^{10}	1.5×10^9	36	22.4	37.3	-3.7%	0.96
1.3×10^{12}	2.1×10^{11}	25	15.0	33.0	-2.6%	0.76
1.2×10^{15}	3.0×10^{14}	9	4.32	26.8	-0.5%	0.34

Table 1: Unification at the scale v in the step-function approximation in the supersymmetric case.

with the threshold corrections δ_i included, all α_i take a common value at the scale $M_{\rm GUT}$ we find that the KK mass levels are the same for the gauge bosons of each group: $M_n^{(1)} = M_n^{(2)} = M_n^{(3)}$. Therefore it is useful to define:

$$F_N \equiv \ln\left(\frac{M_{\rm GUT}^{N-1}}{M_1 \cdots M_{N-1}}\right) = \ln\left[\frac{2^{N-1}}{\sqrt{N}} \sin^{N-1}\left(\frac{(N-1)\pi}{2N}\right)\right] \tag{3.6}$$

We see that eqs. (3.1), after substituting $\delta_i \to \delta_i - \frac{\tilde{b}^{(i)}}{2\pi} F_N$, become identical to eqs. (3.5). In the present case (which is qualitatively the same as the case of extra dimensions [8, 9]) we have a new source of threshold corrections, with the function F_N uniquely determined by the number of replications of the SM(MSSM) gauge group. The condition for unification is now:

$$\epsilon_{ijk}\left(\frac{1}{\tilde{\alpha}_i(M_Z)} + \delta_i - \frac{\tilde{b}^{(i)}}{2\pi} F_N\right)\left(b_0^{(j)} - b_0^{(k)}\right) = 0 \tag{3.7}$$

Putting in the experimental numbers and the coefficients of the beta-functions we obtain:

$$\begin{aligned} -41.1 + 3.8\delta_1 - 11.1\delta_2 + 7.3\delta_3 + 0.01F_N = 0 \quad &\text{(SM)} \\ -0.9 + 4\delta_1 - 9.6\delta_2 + 5.6\delta_3 + 0.38F_N = 0 \quad &\text{(MSSM)} \end{aligned} \tag{3.8}$$

Let us investigate the impact of F_N on the unification. With $\delta_1 = \delta_2 = 0$, the unification scale can be determined from the equation:

$$\frac{1}{\tilde{\alpha}_1(M_Z)} - \frac{1}{\tilde{\alpha}_2(M_Z)} - \frac{1}{2\pi}\left(b_0^{(1)} - b_0^{(2)}\right)\ln\left(\frac{M_{\rm GUT}}{M_Z}\right) - \frac{1}{2\pi}(\tilde{b}^{(1)} - \tilde{b}^{(2)})F_N = 0 \tag{3.9}$$

The results in the supersymmetric case are summarized in the table 1 in which the required threshold correction δ_3 is also given. It is clearly seen that in order to lower the unification scale significantly one needs large-N. We observe also that even for the unification scale as low as 10 TeV the required threshold corrections are of order 6%. However, for unification scales below $\sim 10^{10}$ GeV, the effective loop expansion parameter $\alpha \equiv N\tilde{\alpha}$ becomes larger than 1 which disfavours such a possibility.

In the non-supersymmetric case the threshold corrections are little sensitive to the size of F_N because of the small coefficient in front of F_N in eq. (3.8). Thus, we can lower the unification scale down to ~ 10 TeV and still maintain the threshold corrections of the order of 10% $\alpha_{\rm GUT}^{-1}$. The comment on large-N also applies in this case.

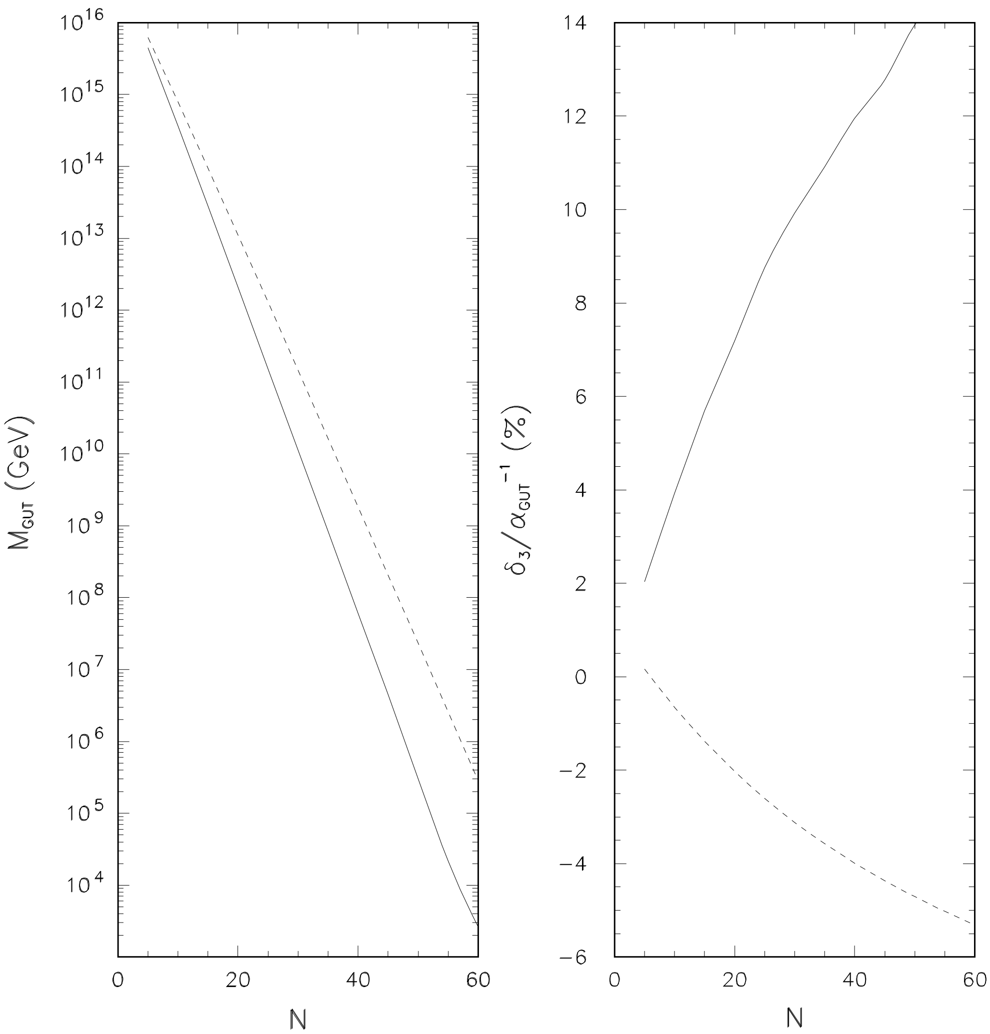

Figure 1: M_{GUT} and $\delta_3/\alpha_{\mathrm{GUT}}^{-1}$ (in %) as functions of N for replicated MSSM. Solid lines show the results of the numerical integration of the full mass-dependent beta-functions. Dashed lines correspond to the step-function approximation.

Given the large number of the heavy vector boson thresholds it is interesting to go beyond the step-function approximation and to investigate, by means of the numerical integration, the running with the full mass-dependent beta-functions (A.3). Although these mass effects are formally of higher order, one can check if they are not enhanced by the accumulation of thresholds. The results of the numerical integration are presented in figure 1 (solid lines) and compared with those obtained in the step-function approximation. As could be expected, the difference between the two results increases with N but we see, that the full mass effects are indeed of the same order of magnitude as the two-loop effects in the MSSM. Typical evolution of (inverse) gauge couplings in the models with replicated gauge groups is shown in figure 2a and b for $v = 10^5\,\mathrm{GeV}$, $N = 60$ and $v = 10^{10}\,\mathrm{GeV}$, $N = 32$, respectively.

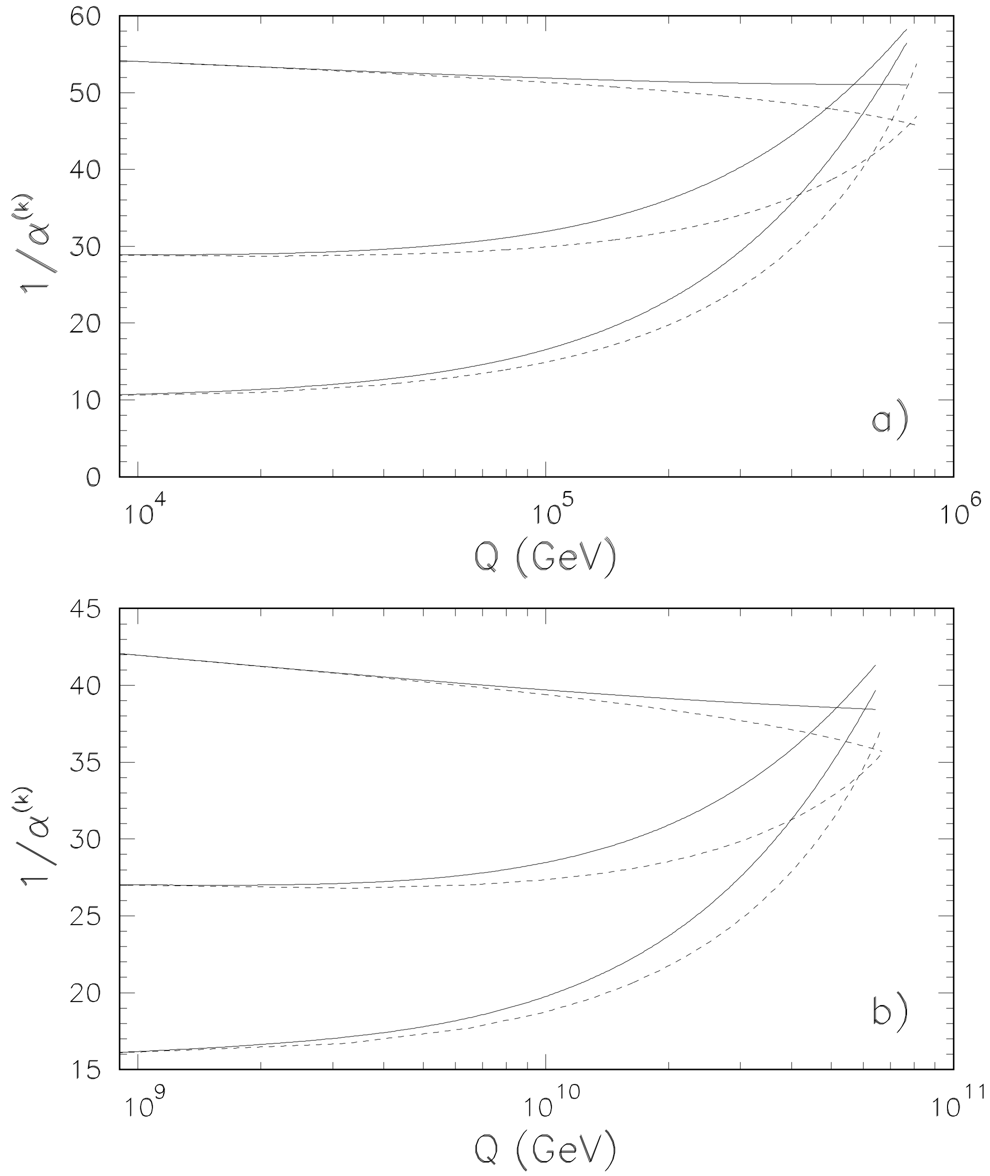

Figure 2: Evolution of (inverse) gauge couplings in replicated MSSM for $v = 10^5\,\text{GeV}$, $N = 60$ (panel a) and $v = 10^{10}\,\text{GeV}$, $N = 32$ (panel b). Solid lines show the results of the integration of the mass-dependent beta-functions. Dashed lines show the results obtained in the step-function approximation.

Concluding this section, in models with N-fold replication of the SM gauge and Higgs structures unification at the scale of symmetry breaking to the diagonal subgroup requires as large threshold corrections as the unification in the SM itself, $O(10\%)$. The threshold corrections are minimized only for strongly correlated values of v and N. The latter remains true also for the replication of the MSSM. In this case the minimal threshold corrections required at one-loop level and in the step-function approximation remain small and for $M_{GUT} = 10^{10}\,\text{GeV}$ one needs $\delta_3 \sim 4\%$ for a succesful unification. The impact of the full

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mass dependent beta-functions is non-negligible, but remains of the order of the known two-loop corrections in the MSSM. Thus, threshold effects of the massive gauge bosons should be considered on equal footing with two-loop corrections and other subdominant effects (e.g. corrections to the initial values of the couplings in the mass dependent renormalization scheme) in a more precise study of unification. The unification scale below 10^{10} GeV requires very large-N, so that the effective loop expansion parameter $N\tilde{\alpha}$ becomes larger than one.

4. Comparison with unification above the scale of symmetry breaking

It is worth comparing the results of the previous section with unification above the scale of symmetry breaking proposed in ref. [10]. For the sake of comparison we briefly recall the results of ref. [10] and then present some numerical results obtained with the model of section 2 and within the same calculational framework as in section 3.

If, for a fixed v, the number of replications N is smaller than the value required for the unification at the scale v, the gauge couplings $\tilde{\alpha}_i$ are different at that scale. However, above the scale v, the full symmetry group $[G_{SM}]^N$ is restored. In our model, the gauge couplings $\alpha_i^{(n)}$ of all N SM group factors have a common value α_i at the scale of the heaviest gauge boson $M_{N-1}^{(i)}$ and satisfy the relation:

$$\alpha_i(M_{N-1}^{(i)}) = N\tilde{\alpha}_i(M_{N-1}^{(i)}) \tag{4.1}$$

in which $\tilde{\alpha}_i(M_{N-1}^{(i)})$ are determined by the running of gauge couplings below the diagonal breaking scale. Of course, now $\alpha_i(M_{N-1}^{(i)}) \neq \alpha_j(M_{N-1}^{(j)})$ for $i \neq j$. Recall that in the step-decoupling approximation:

$$\frac{1}{\tilde{\alpha}_i(M_{N-1}^{(i)})} = \frac{1}{\tilde{\alpha}_i(M_Z)} - \frac{b_0^{(i)}}{2\pi}\ln\left(\frac{M_{N-1}^{(i)}}{M_Z}\right) - \frac{\tilde{b}^{(i)}}{2\pi}F_N \tag{4.2}$$

Above the scale $M_{N-1}^{(i)}$ each of the gauge couplings $\alpha^{(i)}$ runs as in an ordinary 4d theory. Treating all gauge, matter and Higgs fields (including the link-Higgs fields) as massless above that scale we have:

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(M_{N-1}^{(i)})} - \frac{\tilde{b}_0^{(i)}}{2\pi}\ln\left(\frac{Q}{M_{N-1}^{(i)}}\right) + \delta_i \qquad (Q > M_{N-1}^{(i)}) \tag{4.3}$$

The beta-function coefficients of all but the first gauge group G_{SM} are $\tilde{b}_0 = (53/30, -11/2, -28/3)$ in the non-supersymmetric case and $\tilde{b}_0 = (28/5, 0, -4)$ in the supersymmetric case and include the contribution from the bifundamental Higgs fields.[5] Combining eqs. (4.1), (4.2) and (4.3) we get:

$$\frac{1}{\alpha_i(Q)} = \frac{1}{N}\left(\frac{1}{\alpha_i(M_Z)} - N\frac{\tilde{b}_0^{(i)}}{2\pi}\ln\left(\frac{Q}{M_{N-1}^{(i)}}\right) - \frac{b_0^{(i)}}{2\pi}\ln\left(\frac{Q}{M_Z}\right) - \frac{\tilde{b}^{(i)}}{2\pi}F_N + N\delta_i\right) \tag{4.4}$$

[5]The coefficient of the beta-function of the first G_{SM} factor gets, in addition, the contribution from the (MS)SM matter fields.

To obtain a transparent formula for the unification scale we need to assume that the heaviest gauge boson masses corresponding to all the gauge factors are approximately equal $M^{(1)}_{N-1} = M^{(2)}_{N-1} = M^{(3)}_{N-1} \equiv M_{N-1}$ (in numerical calculations, whose results are shown in the figure 3, they usually differ by a factor of $2-5$). Then assuming $\alpha_1(M_{\rm GUT}) = \alpha_2(M_{\rm GUT}) = \alpha_3(M_{\rm GUT}) \equiv \alpha$ we get the equations:

$$\begin{aligned}
\frac{1}{\tilde\alpha_1(M_Z)} - \frac{1}{\tilde\alpha_2(M_Z)} - \frac{1}{2\pi}(b_0^{(1)}-b_0^{(2)}) \ln\left(\frac{M_{\rm U}^N}{M_{N-1}^{N-1}M_Z}\right) - \frac{1}{2\pi}(\tilde b^{(1)} - \tilde b^{(2)})F_N + N(\delta_1-\delta_2) &= 0\\
\frac{1}{\tilde\alpha_2(M_Z)} - \frac{1}{\tilde\alpha_3(M_Z)} - \frac{1}{2\pi}(b_0^{(2)}-b_0^{(3)}) \ln\left(\frac{M_{\rm U}^N}{M_{N-1}^{N-1}M_Z}\right) - \frac{1}{2\pi}(\tilde b^{(2)} - \tilde b^{(3)})F_N + N(\delta_2-\delta_3) &= 0
\end{aligned} \tag{4.5}$$

We have used the fact that $\tilde b_0^{(i)} - \tilde b_0^{(j)} = b_0^{(i)} - b_0^{(j)}$ which follows from the fact that the bifundamental link-Higgs fields form complete representations of SU(5). These equation have the same form as in the case of unification at the scale v but now [10]:

$$M_{\rm GUT} \to \frac{M^N_{\rm GUT}}{M^{N-1}_{N-1}} \tag{4.6}$$

We also have $\delta_k \to N\delta_k$. This means that threshold corrections needed for succesful unification, in absolute numbers, may be much smaller. But we always express the threshold corrections in the units of α^{-1} at the GUT scale which is also approximetely N-times smaller than in the case of unification at the scale v.

In figure 3 we present the results of numerical calculations in the step-decoupling approximation (left panels) and using full mass dependent beta functions (right panels). As expected from the approximate eq. (4.5), for a chosen value of v, the gauge couplings do unify for any number N of replications. No correlation between v and N is needed and, in this sense, unification of the gauge couplings is the prediction of the model to the same extent as in the MSSM itself, but with N dependent $M_{\rm GUT}$. A small number of replications is sufficient for reaching unification at low scale (this requires low v; of course v is constrained from below, $v \gtrsim 1$ TeV, by experimental limits on the existence of additional massive vector bosons). For not too large value of N, say $N < 10$, the precision of one-loop unification is better than for the unification at the diagonal breaking scale and is almost as satisfactory as in the MSSM. The effects of the full mass dependent beta-functions are very small . Moreover, for such small N, $\alpha_{\rm GUT}$ stays well within the perturbative region. For large-N the couplings unify just above the diagonal breaking scale and the unification scale and the threshold corrections are similar as in the case discussed in the previous section.

5. Conclusions

We have discussed the evolution of the gauge couplings in renormalizable gauge theories based on N-fold replication of the SM(MSSM) gauge groups, spontaneously broken at some scale v to the diagonal subgroup identified with the SM(MSSM) group. A systematic one-loop calculation has been performed below and above the scale v.

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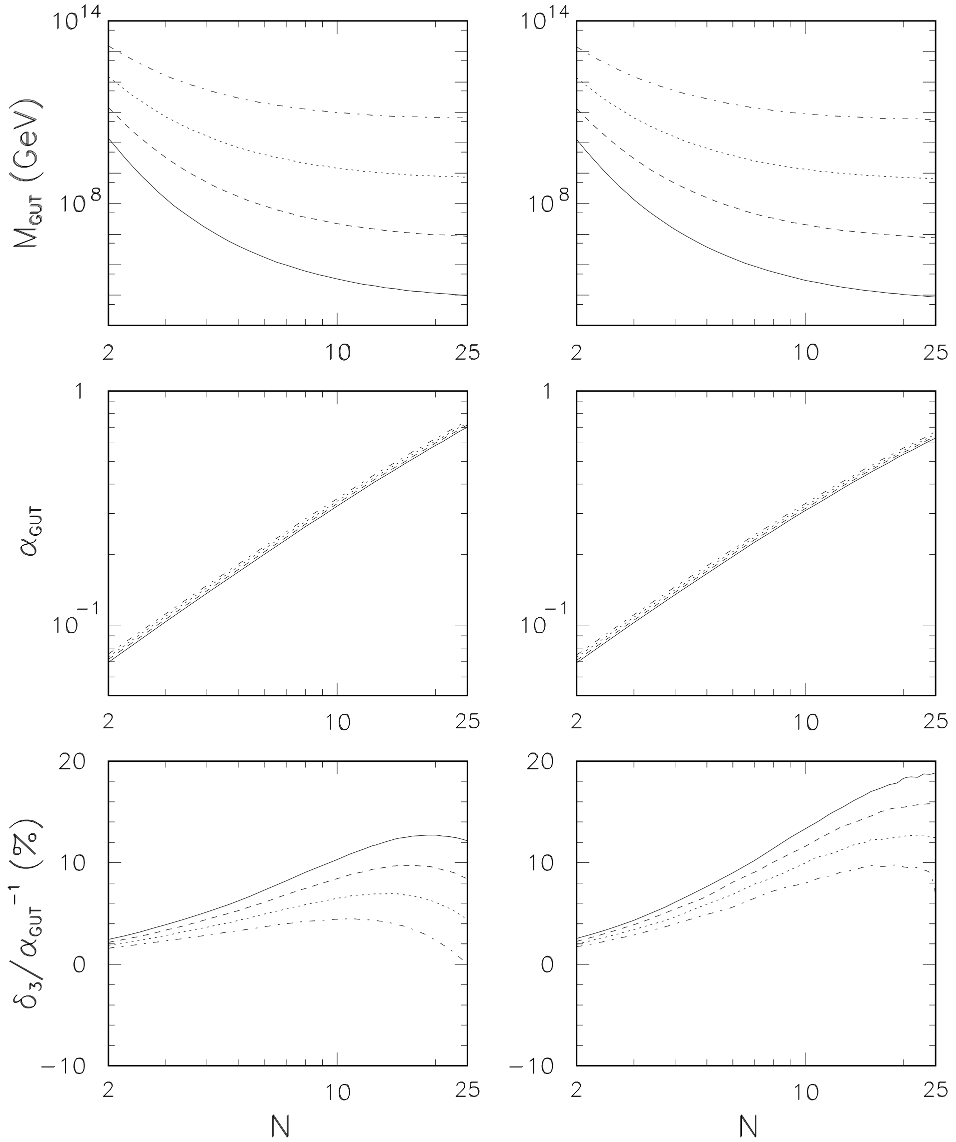
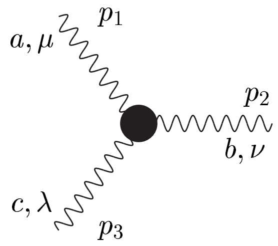


Figure 3: $M_{\rm GUT}$, $\alpha_{\rm GUT}$ and the GUT scale threshold corrections $\delta_3/\alpha_{\rm GUT}^{-1}$ as functions of N for the unification above the diagonal breaking scale in the replicated MSSM. Left (right) panels show the results in the step-function approximation (with the mass-dependent beta functions). Various lines correspond to $v = 10^4$(solid),10^6(dashed),10^8(dotted),10^{10} GeV (dash-dotted).

Unification at the diagonal breaking scale v is possible down to $v \sim 10^5$ GeV; for any value of v there exists a narrow range of N, that gives unification at that scale. This result has the obvious counterpart in the power-law unification in five dimensions [8, 9].

Proposed in ref. [10] unification above the diagonal breaking scale is a new feature of the considered class of models, with no counterpart in theories with extra dimensions. It is particularly attractive as it does not require any correlation between v and N. It is a prediction of the model to the same extent as unification in the MSSM and is a consequence of the latter.



$$V(p_1, p_2) f^{abc} g_{\mu\nu\rho} + \text{ other Lorentz structures}$$
$$g_{\mu\nu\rho} = g_{\mu\nu}(p_1 - p_2)_\rho + g_{\nu\rho}(p_2 - p_3)_\mu + g_{\rho\mu}(p_3 - p_1)_\nu$$

Figure 4: 1-PI three-gluon vertex.

The accuracy of one-loop unification in the considered class of models, particularly of unification above the diagonal breaking scale and with not too large number of replications, is as satisfactory as in the MSSM. Of course, low unification scale raises such problems as the proton decay, but they do not rule out the idea of low scale unification [8, 17].

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A. Beta-functions in models with replicated gauge groups

In this appendix we specify our method of computing the RGE running of the gauge couplings and give the formulae for the full mass-dependent beta-functions which automatically take into account threshold effects due to the massive gauge bosons. For the unbroken non-abelian group factors it is convenient to define the scale dependent gauge couplings as the scalar factor of the full trilinear massless vector boson vertex.

At one-loop the beta-functions are given by the expression:

$$\beta(q^2) = q^2 \frac{\partial}{\partial q^2} \left(2V(q^2) - 3g\Pi(q^2)\right) \tag{A.1}$$

In eq. (A.1) $V(q^2)$ is the sum of one-loop contributions to the vertex defined in figure 4 evaluated at euclidean external momenta $p_1^2 = p_2^2 = p_3^2 = -q^2 < 0$, and $\Pi(q^2)$ is given by the sum of the one-loop vacuum polarization diagrams contributing to the vector boson self-energy $-i(g_{\mu\nu}p^2 - p_\mu p_\nu)\Pi(p^2)$, also evaluated at $p^2 = -q^2 < 0$.

The vector boson self energy gets the contributions summarized in table 2.

The vertex correction $V(q^2)$ gets the contributions summarized in table 3.

Using the above formulae we can easily calculate the contribution of the various fields to the mass dependent beta-function and the results are given in table 4. $A_\mu^{(0)}$,$A_\mu^{(n)}$,ϕ,ψ denote massless gauge boson, massive gauge boson, real scalar field and Weyl fermion, respectively.

Massless gauge boson	$\Pi = g^2 \frac{C_2(G)}{(4\pi)^2} \left(\frac{5}{3} B_0(m,m) - \frac{1}{9}\right)$
Massive gauge boson	$\Pi = g^2 \frac{C_2(G)}{(4\pi)^2} \left(\frac{3}{2} B_0(m,m) - 2(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,m) + \frac{m^2}{q^2})\right)$
Real scalar	$\Pi = g^2 \frac{C_2(r)}{(4\pi)^2} \left(-\frac{1}{6} B_0(m,m) - \frac{2}{3}(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,m) + \frac{m^2}{q^2}) + \frac{1}{9}\right)$
Weyl fermion	$\Pi = g^2 \frac{C_2(r)}{(4\pi)^2} \left(-\frac{2}{3} B_0(m,m) + \frac{4}{3}(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,m) + \frac{m^2}{q^2}) - \frac{4}{9}\right)$

Table 2: Contributions to the self-energy of the massless non-abelian gauge boson.

Massless gauge boson	$V = g^3 \frac{C_2(G)}{(4\pi)^2} (\frac{2}{3} B_0(m,m) - \frac{1}{36} q^2 C_0(m,m,m) - \frac{31}{12})$
Massive gauge boson	$V = g^3 \frac{C_2(G)}{(4\pi)^2} \left(\frac{1}{2} B_0(m,m) - 2m^2 C_0(m,m,m) + \frac{1}{12} q^2 C_0(m,m,m) - \frac{33}{12}\right)$
Real scalar	$V = g^3 \frac{C_2(r)}{(4\pi)^2} \left(-\frac{1}{6} B_0(m,m) - \frac{1}{3} m^2 C_0(m,m,m) + \frac{1}{9} q^2 C_0(m,m,m) - \frac{1}{6}\right)$
Weyl fermion	$V = g^3 \frac{C_2(r)}{(4\pi)^2} \left(-\frac{2}{3} B_0(m,m) + \frac{2}{3} m^2 C_0(m,m,m) + \frac{8}{9} q^2 C_0(m,m,m) - \frac{2}{3}\right)$

Table 3: Contributions to the vertex correction of the massless non-abelian gauge boson.

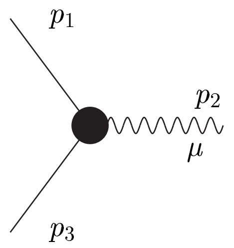
$A_\mu^{(0)}$	$\beta = g^3 \frac{C_2(G)}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2} \left(-\frac{11}{3} B_0(m,m)\right)$
$A_\mu^{(n)}$	$\beta = g^3 \frac{C_2(G)}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2} \Big(-\frac{7}{2} B_0(m,m) + 6\left(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,m) + \frac{m^2}{q^2}\right) + $ $+ \left(-4m^2 + \frac{1}{6} q^2\right) C_0(m,m,m)\Big)$
ϕ	$\beta = g^3 \frac{C_2(r)}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2} \Big(+\frac{1}{6} B_0(m,m) + 2\left(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,m) + \frac{m^2}{q^2}\right) + $ $+ \left(-\frac{2}{3} m^2 + \frac{2}{9} q^2\right) C_0(m,m,m)\Big)$
ψ	$\beta = g^3 \frac{C_2(r)}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2} \Big(+\frac{2}{3} B_0(m,m) - 4\left(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,m) + \frac{m^2}{q^2}\right) + $ $+ \left(\frac{4}{3} m^2 + \frac{8}{9} q^2\right) C_0(m,m,m)\Big)$

Table 4: Contributions to the beta-function of the massless non-abelian gauge boson.

$C_2(r)$ is the quadratic Casimir in the representation r, by G we denote the adjoint representation. Recall that $C_2(G) = N$ for the SU(N) group and $C_2(\mathbf{N}) = 1/2$ for the fundamental representation of SU(N). The functions A, B_0, C_0 are defined as in [16]:

$$\begin{aligned}
\frac{1}{(4\pi)^2} A(m) &= \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \\
\frac{1}{(4\pi)^2} B_0(p_1^2, m_1, m_2) &= \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2 - m_1^2][(k+p_1)^2 - m_2^2]} \\
\frac{1}{(4\pi)^2} C_0(p_1, p_2, m_1, m_2, m_3) &= \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2 - m_1^2][(k+p_1)^2 - m_2^2][(k+p_1+p_2)^2 - m_3^2]}
\end{aligned}$$

In the formulae for the beta functions the above functions are evaluated at $p_1^2 = p_2^2 = -2p_1p_2 = -q^2$.



$-iV(p_1, p_2)Y\gamma_\mu +$ other Lorentz structures

Figure 5: 1-PI U(1) vertex.

Massless gauge boson	$\Sigma = g^2 \frac{Y^2}{(4\pi)^2} \left(-B_0(0,0) - 1\right)$
Massless gaugino	$\Sigma = g^2 \frac{Y^2}{(4\pi)^2} \left(-B_0(0,0) - 1\right)$
Massive gauge boson	$\Sigma = g^2 \frac{\kappa^2 Y^2}{(4\pi)^2} \left(-B_0(m,0) - 1 - \left(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,0)\right)\right)$
Massive gaugino	$\Sigma = g^2 \frac{\kappa^2 Y^2}{(4\pi)^2} \left(-B_0(m,0) - 1 + \left(\frac{A(m)}{q^2} - \frac{m^2}{q^2} B_0(m,0)\right)\right)$

Table 5: Contributions to the fermion self-energy.

It is wortwhile to show that far above and far below the mass threshold our beta functions reduce to their MS scheme counterparts. For $-q^2 \gg m^2$ we need the asymptotic expansions: $A(m) = m^2(-\eta - 1 + \ln(m^2))$, $B_0(m,m) = -\eta + \ln(p^2) + \mathcal{O}(m^2/q^2)$, $q^2 C_0(m,m,m) \sim \text{const} + \mathcal{O}(m^2/q^2)$ (where $\eta = \frac{2}{4-d} + \ln(4\pi) - \gamma_E$). Thus, up to $\mathcal{O}(m^2/q^2)$ corrections, far above threshold the beta function is given by the coefficients multiplying the $B_0(m,m)$ function. The same coefficient multiplies the $\frac{2}{4-d}$ singularity which yields the beta function in the MS scheme. For $-q^2 \ll m^2$ we need $B_0(m,m) = -\eta - 1 + \ln(m^2) + \mathcal{O}(q^2/m^2)$ and $C_0 \sim \mathcal{O}(q^2/m^4)$. It is straightforward to see that all terms of order $\mathcal{O}(m^2/q^2)$ and $\mathcal{O}(1)$ cancel, thus far below threshold the beta function is vanishing up to $\mathcal{O}(q^2/m^2)$ corrections.

For the U(1) factor we of course do not have the trilinear gauge boson couplings. In this case it is convienient to define the scale dependent gauge coupling as the scalar factor of the full vertex with two fermions (e.g. electrons).

The one-loop beta function is given by:

$$\beta(q^2) = q^2 \frac{\partial}{\partial q^2} \left(2V(q^2) - 2g\Sigma(q^2) - g\Pi(q^2)\right) \tag{A.2}$$

In eq. (A.2) $V(q^2)$ is the sum of one-loop contributions to the vertex defined in figure 5, $\Pi(q^2)$ is the the vector boson self-energy and $\Sigma(q^2)$ is the fermion self-energy; all diagrams being evaluated at euclidean external momenta $p_1^2 = p_2^2 = p_3^2 = -q^2 < 0$. Y is the SU(5) normalized hypercharge of the fermion.

The contributions to the U(1) gauge boson self-energy are similar to the non-abelian case, with the group factors $(C_2(G), C_2(r))$ replaced by $(0, Y_r^2)$, respectively. The contributions to the fermion self-energy are summarized in table 5.

Massless gauge boson	$V = g^3 \frac{Y^2}{(4\pi)^2}\left(-B_0(0,0) - 1 + \frac{1}{3}p^2 C_0(0,0,0)\right)$
Massless gaugino	$V = g^3 \frac{\kappa^2 Y^2}{(4\pi)^2}\left(-B_0(0,0) + 1 + \frac{2}{3}p^2 C_0(0,0,0))\right)$
Massive gauge boson	$V = g^3 \frac{\kappa^2 Y^2}{(4\pi)^2}\left(-\frac{4}{3}B_0(m,0) + \frac{1}{3}B_0(0,0) - 1 + \frac{1}{3}p^2 C_0(m,0,0))\right)$
Massive gaugino	$V = g^3 \frac{\kappa^2 Y^2}{(4\pi)^2}\Big(-\frac{2}{3}B_0(m,0) - \frac{1}{3}B_0(0,0) + 1 + \, + \left(\frac{2}{3}p^2 - 2m^2\right) C_0(m,0,0)\Big)$

Table 6: Contributions to the vertex correction of the massless abelian gauge boson.

$A_\mu^{(0)}$	$\beta = 0$
$\chi^{(0)}$	$\beta = 0$
$A_\mu^{(n)}$	$\beta = g^3 \frac{Y^2\kappa^2}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2}\Big(-\frac{2}{3}B_0(m,0) + \frac{2}{3}B_0(0,0) + 2\left(\frac{A(m)}{q^2} - \frac{m^2}{q^2}B_0(m,0)\right) + \, + \frac{2}{3}q^2 C_0(m,0,0)\Big)$
$\chi^{(n)}$	$\beta = g^3 \frac{Y^2\kappa^2}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2}\Big(\frac{2}{3}B_0(m,0) - \frac{2}{3}B_0(0,0) - 2\left(\frac{A(m)}{q^2} - \frac{m^2}{q^2}B_0(m,0)\right) + \, + (\frac{4}{3}q^2 - 4m^2) C_0(m,0,0)\Big)$
ϕ	$\beta = g^3 \frac{Y_r^2}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2}\left(+\frac{1}{6}B_0(m,m) + \frac{2}{3}\left(\frac{A(m)}{q^2} - \frac{m^2}{q^2}B_0(m,m) + \frac{m^2}{q^2}\right)\right)$
ψ	$\beta = g^3 \frac{Y_r^2}{(4\pi)^2} q^2 \frac{\partial}{\partial q^2}\left(+\frac{2}{3}B_0(m,m) - \frac{4}{3}\left(\frac{A(m)}{q^2} - \frac{m^2}{q^2}B_0(m,m) + \frac{m^2}{q^2}\right)\right)$

Table 7: Contributions to the beta-function of the massless abelian gauge boson.

The factors $\kappa = \sqrt{2}cos[\frac{n\pi}{2N}]$ arise because the massive gauge bosons at different mass levels have different couplings to the matter fields. Y_r is the SU(5) normalized hypercharge of the (scalar/Weyl fermion) flying in the loop. The contributions to the 1PI vertex corrections are summarized in table 6.

Finally, in table 7 we present the contributions of various fields to the mass dependent beta-functions of the U(1) gauge coupling. $A_\mu^{(0)}$,$\chi^{(0)}$,$A_\mu^{(n)}$,$\chi^{(n)}$,ϕ,ψ denote massless gauge boson, massless gaugino, massive gauge boson, massive gaugino, real scalar field and Weyl fermion, respectively.

Now we are ready to write the expressions for the full beta-functions in the supersymmetric version of our model. Apart from the (approximately massless) MSSM spectrum with N_g generations we have a tower of N-1 gauge multiplets of $\mathcal{N} = 2$ supersymmetry consisting of a vector boson, two Weyl fermions and a scalar, all in the adjoint represantion of the SM gauge group. The members of this multiplet have masses given by $M_n^{(i)} = 2g(v)v\sin(n\pi/2N)$. There are also $\tilde{N}_h$ towers of $(N-1)$ SU(2) doublet chiral supermultiplets (a scalar and a Weyl fermion) which for simplicity at every level are assumed to have the same mass as the SU(2) massive supermultiplet.

The beta-functions turn out to be:

$$
\begin{aligned}
\beta^{(k)}(q^2) &= g^3 \frac{1}{(4\pi)^2}\left(b_0^{(k)} + q^2 \frac{\partial}{\partial q^2}\sum_{n=1}^{N-1} \tilde{b}^{(k)}(M_n^{(k)})\right) \\
b_0^{(3)} &= -9 + 2N_g \\
b_0^{(2)} &= -5 + 2N_g \\
b_0^{(1)} &= \frac{3}{10} + 2N_g \\
\tilde{b}^{(3)}(m) &= -6B_0(m,m) - 6m^2 C_0(m,m,m) + \frac{13}{2} q^2 C_0(m,m,m) \\
\tilde{b}^{(2)}(m) &= \left(-4 + \frac{1}{2}\tilde{N}_h\right) B_0(m,m) - 4m^2 C_0(m,m,m) + \left(\frac{13}{3} + \frac{2}{3}\tilde{N}_h\right) q^2 C_0(m,m,m) \\
\tilde{b}^{(1)}(m) &= \frac{3}{10}\tilde{N}_h B_0(m,m) + \kappa^2(2q^2 - 4m^2) C_0(m,0,0)
\end{aligned}
\tag{A.3}
$$

In the step-decoupling approximation the contribution to the beta coefficients from the masive modes is given by the coefficient multiplying B_0 for $-q^2 > m^2$ and is vanishing for $-q^2 < m^2$.

References

[1] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *(De)constructing dimensions*, *Phys. Rev. Lett.* **86** (2001) 4757.

[2] C.T. Hill, S. Pokorski and J. Wang, *Gauge invariant effective lagrangian for Kaluza-Klein modes*, *Phys. Rev.* **D 64** (2001) 105005 [hep-th/0104035].

[3] C. Csáki, G.D. Kribs and J. Terning, *4D models of Scherk-Schwarz SUSY breaking via deconstruction*, *Phys. Rev.* **D 65** (2002) 015004 [hep-ph/0107266].

[4] H.-C. Cheng, K.T. Matchev and J. Wang, *SUSY breaking on the lattice*, *Phys. Lett.* **B 521** (2001) 308 [hep-ph/0107268].

[5] E.C. Poggio, H.R. Quinn and S. Weinberg, *Smearing the quark model*, *Phys. Rev.* **D 13** (1976) 1958.

[6] D.A. Ross, *Threshold effects in gauge theories*, *Nucl. Phys.* **B 140** (1978) 1.

[7] T.R. Taylor and G. Veneziano, *Strings and $D = 4$*, *Phys. Lett.* **B 212** (1988) 147.

[8] K.R. Dienes, E. Dudas and T. Gherghetta, *Extra spacetime dimensions and unification*, *Phys. Lett.* **B 436** (1998) 55 [hep-ph/9803466]; *Grand unification at intermediate mass scales through extra dimensions*, *Nucl. Phys.* **B 537** (1999) 47 [hep-ph/9806292].

[9] H.-C. Cheng, B.A. Dobrescu and C.T. Hill, *Gauge coupling unification with extra dimensions and gravitational scale effects*, *Nucl. Phys.* **B 573** (2000) 597.

[10] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *Accelerated unification*, hep-th/0108089.

[11] C. Csáki, J. Erlich, C. Grojean and G.D. Kribs, *4d constructions of supersymmetric extra dimensions and gaugino mediation*, *Phys. Rev.* **D 65** (2002) 015003 [hep-ph/0106044].

[12] Particle data group, C. Caso, *et al*, *Review of particle physics. Particle data group*, *Eur. Phys. J.* **C 3** (1998) 1.

JHEP08(2002)003

[13] J. Hisano, H. Murayama and T. Yanagida, *Nucleon decay in the minimal supersymmetric* SU*(5) grand unification*, *Nucl. Phys.* **B 402** (1993) 46 [hep-ph/9207279];
J. Bagger, K.T. Matchev and D. Pierce, *Precision corrections to supersymmetric unification*, *Phys. Lett.* **B 348** (1995) 443 [hep-ph/9501277].

[14] J.R. Ellis, S. Kelley and D.V. Nanopoulos, *Probing the desert using gauge coupling unification*, *Phys. Lett.* **B 260** (1991) 131.

[15] P. Langacker and N. Polonsky, *Uncertainties in coupling constant unification*, *Phys. Rev.* **D 47** (1993) 4028 [hep-ph/9210235]; *The strong coupling, unification, and recent data*, *Phys. Rev.* **D 52** (1995) 3081; P.H. Chankowski, Z. Pluciennik, S. Pokorski and C.E. Vayonakis, *Gauge coupling unification in SUSY and string models*, *Phys. Lett.* **B 358** (1995) 264 [hep-ph/9506393]; $\sin^2\theta_W(M_Z)$ *in the MSSM and unification of couplings*, *Nucl. Phys.* **B 439** (1995) 23 [hep-ph/9411233];
S. Pokorski, *Status of the minimal supersymmetric standard model*, in proceedings of the *International workshop on supersymmetry and unification of fundamental interactions*, I. Antoniadis and H. Videau eds., Ecole Polytechnique Palaiseau, France, Mai 1995, hep-ph/9510224;
J. Bagger, K.T. Matchev and D. Pierce, *Precision corrections to supersymmetric unification*, *Phys. Lett.* **B 348** (1995) 443; *Complete weak scale threshold corrections in the minimal supersymmetric standard model*, in proceedings of the US-Polish workshop *Physics from Planck scale to electroweak scale*, P. Nath, T. Taylor and S. Pokorski eds., Warsaw, Poland, September 1994, hep-ph/9501378.

[16] A. Axelrod, *Flavor changing* Z_0 *decay and the top quark*, *Nucl. Phys.* **B 209** (1982) 349.

[17] N. Arkani-Hamed and M. Schmaltz, *Hierarchies without symmetries from extra dimensions*, *Phys. Rev.* **D 61** (2000) 033005 [hep-ph/9903417];
N. Weiner, *Unification without unification*, hep-ph/0106097.

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